-The second configuration is the σ_X coupling

$$H_S = \sum_{\eta=L,R} \varepsilon_\eta \sigma_z^\eta + \kappa \sigma_x^L \sigma_x^R$$

$$R = \frac{|(\Lambda_L + \Lambda_R)(\tanh(\beta_H \kappa/2) - \tanh(\beta_C \kappa/2)|}{|(\Lambda_L - \Lambda_R)(\tanh(\beta_H \kappa/2) + \tanh(\beta_C \kappa/2))|}$$

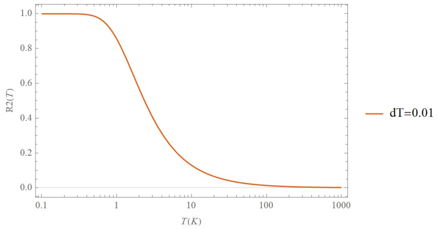

Figure 6-X-coupling rectification.

-The third configuration is the $\sigma^+\sigma^-$ coupling

$$H_S = \sum_{\eta=L,R} \varepsilon_\eta \sigma_z^\eta + J(\sigma_L^+ \sigma_R^- + \sigma_L^- \sigma_R^+)$$

$$R = \frac{|(\Lambda_L + \Lambda_R)(\tanh(\beta_C J/2) - \tanh(\beta_H J/2))|}{|(-\Lambda_L - \Lambda_R)(\tanh(\beta_H J/2) + \tanh(\beta_C J/2))|}$$

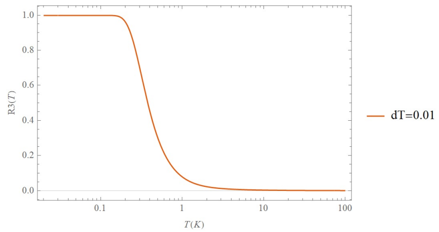

Figure 7-(+/-)-coupling rectification.

At the same fixed parameters, the different couplings between the qubits cause a shift in the average temperature that brings the ratio R to zero, meaning that from a certain T and beyond the system cannot rectify. By using the limit of the zero inner coupling and the Maclaurin series around the temperature difference one can see that, when the difference is zero, the system would never be able to rectify, independently of the parameters involved.

Conclusions

This treatment put the basis for the study of heat management in few-level artificial atoms, which leads to a full amount of possible insights. Once introduced the mathematical instruments suitable for any quantum system in open contact with a larger environment and deduced the Master equations for n-level systems, I reached the proper set of elements and the operational scheme which is used to study these microscopic systems. The two-level system is quite easy to handle from a mathematical point of view and gives the opportunity to see how the rectification behaves depending on an asymmetry factor in the baths' couplings. However, this system reaches just soft rectification, so I decided to study a similar structure by adding more levels to the central system. A three level system granted a larger domain in which rectification is non trivial, but the real change in the rectification strength can be seen in the case of two coupled qubits with different inner couplings. Such a device acts like a heat diode and sustains an high rectification in a noticeable range of temperatures. Furthermore, there is the chance to play with the parameters and see how they can affect the value of the temperature that corresponds to the asymptotic trend to zero of the rectification. These whole results set the conditions to deeply investigate the quantum heat transport in other few-levels systems, perhaps more complex than these ones, and try to figure out how to transform them into real devices.

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MASTER OF SCIENCE IN PHYSICS

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THERMAL RECTIFICATION THROUGH INTERACTING FEW-LEVELS ARTIFICIAL ATOMS

FINAL PROJECT

SUPERVISORS:
PROF.SSA E. PALADINO
PROF. R. FAZIO

ACADEMIC YEAR 2019/2020

Introduction

The rapid development of nanotechnologies have stimulated an extreme jump towards the control of quantum systems, down to the level of single atoms. The implementation of such small systems into modern technologies have also moved the attention to the limited capability of controlling the heat flowing through these devices during the energy transfer processes. One of the possible solutions to this problem is the heat rectification. A thermal current is rectified when it is enhanced in one direction, for a certain bias, and suppressed in the opposite direction, if the bias is reversed. For a microscopic system, the positive and negative terminals are often represented by two heat baths with two different temperatures. Moreover, a quantum heat rectifier is composed by a central quantum system and two reservoirs at its ends.

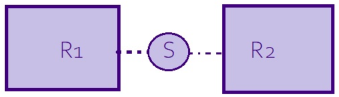

Figure 1-Quantum system communicating with two reservoirs at different temperatures.

In this thesis, I considered a few simple models of interacting few-levels artificial atoms, coupled to two thermal baths at different temperatures.

Background

In order to understand the method used to find the thermal quantities for all the studied systems, it's better to recall some basic concepts. Firstly, the central artificial atom is an open quantum system, always in contact with its surrounding, that can be described by a density matrix with a specific time dependence. The total density matrix, under the justified Markoff approximation can be seen as the product of the system's matrix and the equilibrium thermal distribution of the reservoirs. The solution for the time dependent partial density matrix regarding the system's evolution in the interaction picture, can be found through the derivation of the Master equations

$$\rho(\dot{t})_{mm} = \sum_{n\neq m} \rho(t)_{nn} W_{mn} - \rho(t)_{mm} \sum_{n\neq m} W_{nm}$$

Method

For each system the main procedure consists of: the identification of the Hamiltonian and the density matrices at t=0, the evaluation of the transition rates between the levels involved and the exchanged energies and the solution of the relative Master equation in the stationary case for the populations. Then, it follows the calculation of the thermal currents induced by each bath, depending on the energies, transition rates and populations. To find the rectification ratio of the first two systems I used the definition:

$$R = \frac{|J^+|}{|J^-|}$$

where R=1 means total equilibrium for the positive and negative bias fluxes, R>1 occurs when the positive bias flux overcomes the other, R<1 otherwise. For the last system instead I considered to study the rectification as:

$$R = \frac{|J^+ - J^-|}{|J^+ + J^-|}$$

which means that no rectification occurs when this ratio is 0 and perfect rectification occurs when this is 1.

Qubit rectifier

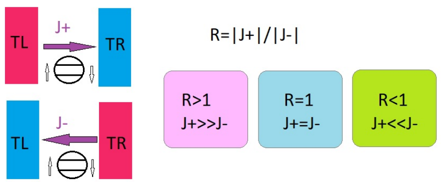

Figure 2-Two level system representation.

The two level system can be represented by the Hamiltonian $H_Q = \frac{\varepsilon}{2}\sigma_z$. After solving the master equations in the stationary case $\rho_{11} = \frac{1}{e^{\varepsilon/kT}+1}$; $\rho_{22} = \frac{e^{\varepsilon/kT}}{e^{\varepsilon/kT}+1}$ one can use this definition for the thermal current $J_{L,R}(\beta_{L,R}) = \varepsilon(\rho_{ii}\Gamma^+_{ij} - \rho_{jj}\Gamma^-_{ji})$ and retrieve the rectification (if one of the baths' temperatures is much higher than the other)

$$R = \frac{\Lambda_R \coth(\beta\varepsilon/2) + \Lambda_L}{\Lambda_L \coth(\beta\varepsilon/2) + \Lambda_R}$$

Qutrit rectifier

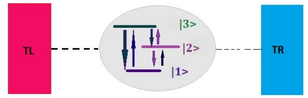

Figure 3-Three level system representation.

$$H_S = (\delta + \gamma) S_z$$

The three level system can be represented by the Hamiltonian $H_Q = \frac{\varepsilon}{2}\sigma_z$. After solving the master equations in the stationary case $\rho_{11} = \frac{1}{e^{2\beta\omega}+e^{\beta\omega}+1}$; $\rho_{22} = \frac{e^{\beta\omega}}{e^{2\beta\omega}+e^{\beta\omega}+1}$; $\rho_{33} = \frac{e^{2\beta\omega}}{e^{2\beta\omega}+e^{\beta\omega}+1}$ one can use the same definition for the currents and retrieve the rectification

$$R = \frac{|\Lambda_L[\frac{e^{2\beta_H\delta}-1}{e^{3\beta_H\delta}-1}] + \Lambda_R[\frac{e^{\beta_C\delta}(e^{2\beta_C\delta}-1)}{e^{3\beta_C\delta}-1}]|}{|\Lambda_L[\frac{e^{\beta_C\delta}(e^{2\beta_C\delta}-1)}{e^{3\beta_C\delta}-1}] + \Lambda_R[\frac{e^{2\beta_H\delta}-1}{e^{3\beta_H\delta}-1}]|}$$

Two coupled qubits rectifier

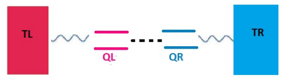

Figure 4-Two coupled qubits representation.

In this case I considered three couplings between two qubits: -The first configuration is the σ_Z coupling

$$H_S = \sum_{\eta=L,R} \varepsilon_\eta \sigma^\eta_z + \xi(\sigma^L_z \otimes \sigma^R_z)$$

$$R = \frac{|(\Lambda_L + \Lambda_R)(\tanh(\beta_C\xi/2) - \tanh(\beta_H\xi/2)|}{|(-\Lambda_L + \Lambda_R)(\tanh(\beta_H\xi/2) + \tanh(\beta_C\xi/2))|}$$

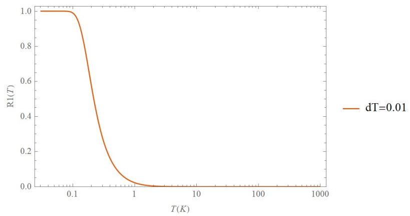

Figure 5-Z-coupling rectification.