and this is noticeable because it is supposed that the periodicities observed about solar activity are due to stochastic fluctuations rather than a nonlinear behavior. That is why we choose time window large enough to reduce the random stochastic noise. Furthermore, the increasing n.o of datapoints acts on the symmetry of the whole trajectory, leading to a passage from an asymmetric pattern to a round shaped object. About the numerical solutions, the parametrical variation of nonlinarity parameter μ and cycle's peak amplitude λ for fixed oscillation frequency ω and cycle's amplitude ξ showed that λ acts over the $min \rightarrow max$ phase of the cycle. The comparison of these results showed that, fixed $\omega \neq 0, \xi \neq 0, \lambda = 0/\neq 0$, the nonlinearity parameter μ assumes a range of values dependent on: the time window's size set for the data-proxy rolling mean and on the presence of λ. If we denote $\mu(WS)$ where capital letters stay for Window Size, we can identify the following range:
$\mu \in [0.15, 0.1 \times 10^{-2}]$
Van der Pol $\mu(155) \geq \mu(455)$
Duffing $\mu(155) \geq \mu(455)$
Van der Pol-Duffing comparison $\mu_V \geq \mu_D$
$\rightarrow \mu\ decreases\ for\ increasing\ WS$, $\mu\ decreases\ for\ \lambda \neq 0$
This means that the peak amplitude λ exerts a weakening action on the cycle's asymmetry, represented by μ.

Conclusions

We observed a dependence of the data-proxy trajectory on the time window choose to perform the rolling mean over the SSN time series. We noted that an average over a time window greater than one year means a reduction of the fluctuations. Moreover, the n.o of datapoints set to build the data-proxy phase space affects the global shape of the cycles. In particular, for a low datapoint number the trajectory results in a threefold pattern which, for increasing n.o of datapoints, evolves till a round shaped trend. About the trajectory of the nonlinear oscillators, we observed that the presence of the λ parameter influences the slope of half a cycle. In the Duffing oscillator's the passage from the min to the max value of the temporal derivative is smoother and symmetrical respect to the Van der Pol's case. Hence, taking into account λ leads to a weakening effect on the cycle's asymmetry. For both the Van de Pol and the Duffing equations, the value of μ ranges between 0.15 and

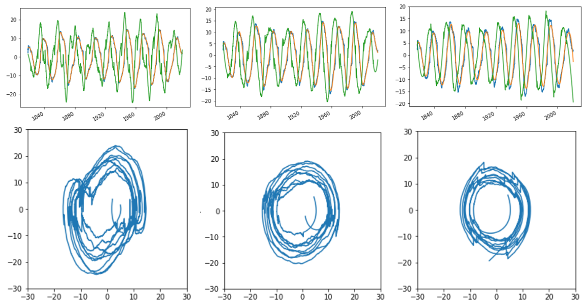

Figura 3: *[Rolling mean time window = 365 days - N.o of datapoints from the left: 7-11-17]Top row: variation of the data-proxy time series and the embedded time sequence. Bottom row: phase space where are plotted $\dot{B}$ vs B*

$0.1x10^{-2}$. Correlating this parametrical interval with the average-dependent data-proxy trajectory, the value of μ is lower if $\lambda \neq 0$, i.e for the Duffing oscillator.

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MASTER OF SCIENCES IN PHYSICS

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NONLINEAR MODELS OF THE SOLAR CYCLE

MASTER THESIS

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Abstract

The solar environment provides a great amount of instrumental challenges and open issues. In this work we focus our attention on the temporal evolution of the solar magnetic field. In particular we deal with a nonlinear dynamo approach which tries to model and explain the detected emicyclic polarity reversal and the global mechanism of solar magnetic field regeneration. We performed a temporal analysis of the toroidal component of the magnetic field projecting the sunspot number (SSN) time series onto a 2D phase space. The key assumption is the proportionality between the SSN indicator and the square of the erupted magnetic field. We found that the time embedding technique is a useful tool in order to investigate the nonlinear behavior of the solar cycle's activity described by the low-order truncated formulation of $\alpha - \Omega$ dynamo equations.

Background

The main issue in studying dynamo mechanism is to find or produce a flow field u dynamically consistent, characterized by inductive properties and able to sustain the magnetic field against the ohmic dissipation. Furthermore, it must be provided the amplification of the the magnetic field B and such a process occurs via flow compression, shearing and transport of the pre-existing magnetic field. The theoretical skeleton consist of a mean field approach with an axisymmetric formulation in spherical coordinates. The total magnetic field is decomposed into $_P$ and B_Φ, expressing the poloidal one through A_P and truncating their equations to confine the solutions to the magnetic field phenomena acting at the bottom of the convective zone. The $\alpha\Omega$ induction equation is expressed in form of two coupled differential equation. Through a parametrical formulation allows a relation between the temporal behavior of the magnetic field B and the numerical solutions of a nonlinear oscillator. The importance of the correspondence between the observational data behavior and nonlinear oscillator solutions lies in the field of solar cycle predictability: it is easy to understand that if the comparison is consistent for the yet observed records, it will be consistent for the expected but unpredicted data. According to dynamo theories, the sunspot number SSN is proportional to the magnetic energy that erupts at the solar surface. Hence it is reasonable to establish the following quantitative relation between the SSN and the magnetic field: $SSN \propto \int B_\Phi^2 dV \rightarrow B_\Phi(t) = \pm\sqrt{SSN} \rightarrow SSN = x^2$

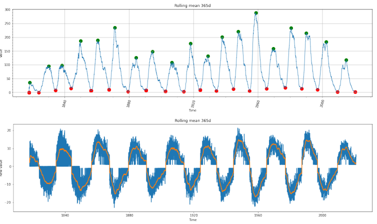

Figura 1: *Top: SSN time series. Bottom: data-proxy*

Objectives

We decided to operate over two routes: the data-proxy trajectory onto the embedded phase space and the numerical solutions of the nonlinear equation. We overlapped both behaviors in virtue of the fact that the SSN is proportional to the magnetic field B which in turn can be identified with the displacement variable x. Hence:

-We decided to employ data-proxy built with a window's size of 155 days and 455 days, corresponding respectively to atmospheric and innermost periodicities.

- We superimposed the trajectory computed for two nonlinear equations: the Van der Pol oscillator, characterized by the absence of λ parameter and the Duffing oscillator, which comprehends the contribution of μ and λ.

We matched both trajectories to investigate the existence of some range of the oscillator's parameters within which the properties observed from the data-proxy cyclic trend are well reproduced.

Techniques

From a quantitative point of view, we attempt to take the square value of the SSN indicator provided by the SSN time series. At the same time we want to manifest a qualitative feature of the erupted magnetic field, namely the polarity reversal, which in turn implies a cyclic trend we must reproduce. Hence, starting from a temporal sequence of maxima and minima daily records we need to perform a rolling average which acts as a filter. Secondly we apply an inversion of the maxima respect to the minima, in order to reproduce the field reversal and we take the square of every value to transform the SSN indicator in the magnetic field intensity. At this point the original SSN time series results substituted by a new sequence we can call data-proxy. In order to study the temporal behavior of the erupted magnetic field respect to the solar cycle activity, we represent the timeseries onto a two-dimensional space where B is plotted on the horizontal axes while dB/dt on the vertical one. Thanks to this representation is possible to have a vision quick and global about the data proxy and infer its main features. This is a crucial aspect because enable us to employ the nonlinear oscillator equation. Clearly the displacement variable x conceptually coincides with B, therefore the numerical solutions of the nonlinear oscillator equation represent the theoretical expected behavior of solar cycle activity. The parameters ω, μ, ξ and λ play a key role because easily evidence and resume the main features of the solar dynamics and, at the same time, comprehend the fundamental processes at the basis of the solar dynamo mechanism.

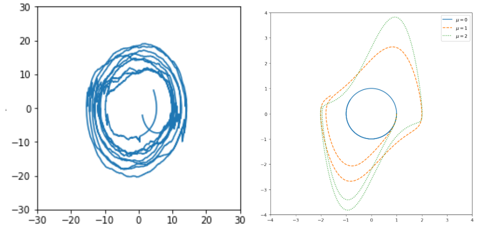

Figura 2: *Left: data-proxy trajectory. Right: nonlinear oscillator numerical solution*

Results

We can divide the whole procedure performed in three blocks: work on the SSN time series, computation of numerical solutions of a nonlinear differential equation and comparison of the resulting trajectories onto the same phase space. The averaging operations over the SSN's data-proxy revealed a dependence on two quantities: the temporal window's size and the number of datapoints. Greater is the time window, weaker are the fluctuations displayed