Charge and spin in a two-dimensional electron gas: always an exciting encounter

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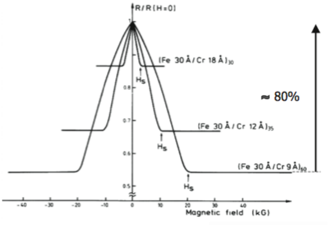
Why Spintronics is interesting?

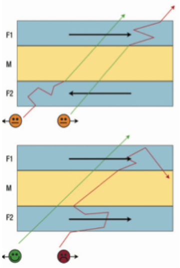
Spintronics: spin transport electronics or spin based electronics
S. A. Wolf, et al. Science **294**, 1488 (2001)

Pesin and MacDonald in Nature Materials (**11**, 409 (2012)) say:

The central goals of spintronics are to understand mechanisms by which it is possible to achieve efficient electrical control of spin currents and spin configurations, and to discover materials in which these mechanisms are prominently exhibited.
Because of the obvious relationship to magnetic information storage technologies, the possibility of applications is always in the background of spintronics research topics, and sometimes jumps to the foreground to spectacular effect.
Nevertheless, the problems that arise in this field are often intriguing from a fundamental point of view, and many topics are pursued for their intrinsic interest.

Classical Spintronics: Discovery of GMR in magnetic multilayers





Schematics of the GMR effect: in the *parallel* configuration, the majority spins can travel easily through all the layers and the short circuit through their channel yields a low resistance. For *antiparallel* configuration, neither channel travels easily.

First observation of giant magnetoresistance in magnetic multilayers Fe/Cr(001) made with molecular beam epitaxy (Baibich et al. PRL **61**, 2472 (1988)).

Without the magnetic field, Brillouin scattering reveals the antiferromagnetic exchange interaction among the magnetic layers (Grünberg et al. PRL **57**, 2442 (1986)).

Nobel Prize 2007: Albert Fert and Peter Grünberg *for the discovery of Giant Magnetoresistance*

EPS's Hewlett- Packard Europhysics Prize 1997: Stuart Parkin *for his pioneering work in GMR technology for hard disks*

Take-home messages

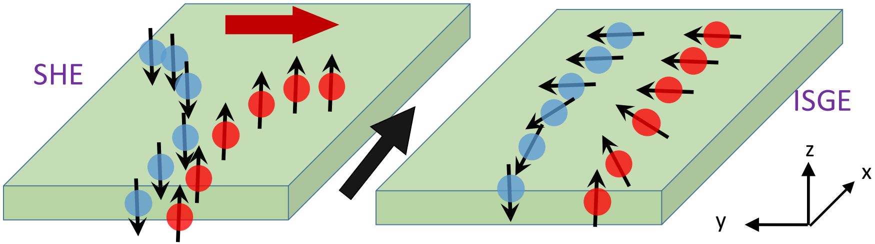
1. Spin-charge conversion mechanisms in transport phenomena are studied in a large variety of systems ranging from semiconductors, metals and insulators. Optical and spin pumping techniques are used together with the standard electric-field induced excitation of carriers.
2. The general features of these phenomena can be understood by appealing to symmetry arguments analyzing in particular time reversal and parity.
3. The microscopic mechanism responsible for spin-charge conversion is the spin-orbit coupling (SOC) and we talk today of Spin-orbitronics. SOC in solid state systems manifests in a number of ways. Its description in the two-dimensional electron gas can be formulated in an elegant way as an effective SU(2) gauge theory.

Outline

1. The spin Hall (SHE) and spin galvanic effects (SGE): definitions and selected experiments
2. General considerations about SOC in solids
3. The Rashba model and its SU(2) gauge theory formulation
4. Extrinsic SOC: Skew scattering, side jump, spin current swapping mechanisms
5. Elliott-Yafet spin relaxation and solution of a paradox
6. Conclusions

For a recent pedagogical exposition see also Raimondi et al. arxiv:1611.07210 (based on Lectures given at The Twelfth International School on Theoretical Physics Symmetry and Structural Properties of Condensed Matter, Poland, Rzeszów, September, 5-10th 2016.)

Spin-charge conversion phenomena



Spin Hall Effect

$$\text{SHE}: \quad J_y^z = \sigma^{SHC} E_x$$

$$\text{ISHE}: \quad J_x = -\sigma^{SHC} E_y^z$$

Spin Galvanic Effect

$$\text{ISGE}: \quad S^y = \sigma^{SGC} E_x$$

$$\text{SGE}: \quad J_x = \sigma^{SGC} \partial_t B^y$$

SHE: history sketch and general arguments (Dyakonov and Perel 1971)

Dyakonov PRL **99**, 126601 (2007)

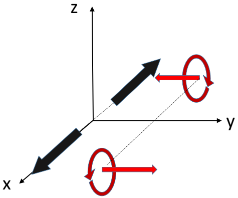
- Charge current: odd under $\mathcal{T}$ and $\mathcal{P}$
- Spin current: odd under $\mathcal{P}$, even under $\mathcal{T}$
- For $\mathcal{P}$-invariant systems Onsager relations imply one parameter γ proportional to spin-orbit coupling (SOC) (Mott skew scattering)

$$\begin{aligned} \mathrm{j}_i &= \sigma E_i - \sigma^a \mathcal{E}_i^a - 4\, \varepsilon_{ija}\, \gamma\, \mathrm{j}_{ja} \\ \mathrm{j}_{ia} &= -\frac{\sigma}{4e^2}\mathcal{E}_i^a + \sigma^a E_i + \varepsilon_{iak}\, \gamma\, \mathrm{j}_k, \end{aligned}$$

Further and more recents suggestions

- Hirsch 1999
- Zhang 2000
- Murakami, Nagaosa, Zhang 2003
- Sinova et al. 2004
- Review: Vignale 2010
- Sinova et al. 2016

SGE: history sketch and general arguments



Ganichev et al. arxiv:1606.02043

In non-centrosymmetric gyrotropic (polar and axial vectors behave similarly) crystals, spin polarization by electrical current, inverse spin-galvanic effect, occurs because of restricted symmetry conditions

$$J_a = Q_{ab} S^y$$

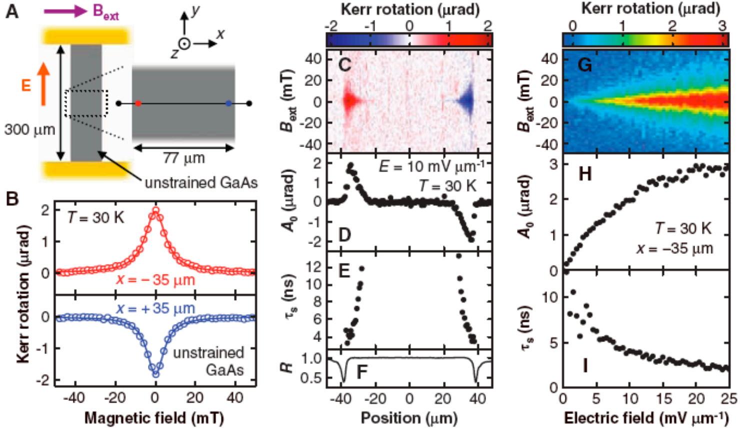
For Quantum Wells under the C_{2v} symmetry, the only non vanishing element of the pseudotensor is Q_{xy}
$Q_{xy} \sim$ SOC

Key papers

- Ivchenko and Pikus 1978
- Levitov, Nazarov and Éliashberg, 1985
- Edelstein 1990
- Aronov and Lyanda-Geller 1989,
- Ganichev et al. 2002
- Review: Ganichev, Trushin, Schliemann 2016
- Review: Ando and Shiraishi 2017
- Review: Soumyanarayanan, Reyren, Fert and Panagopoulos 2017

SHE: Kato et al. Science **306**, 1910 (2004)

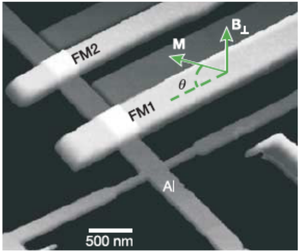
Kerr spectroscopy in GaAs thin film

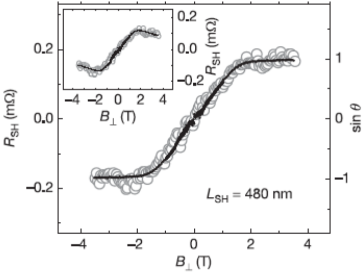


Spin accumulation at edges $\sigma^{SHC} \sim 1\Omega^{-1}m^{-1}$

ISHE: Valenzuela and Tinkham, Nature **442**, 176 (2006)

Metallic system: Ferromagnetic permalloy in contact with aluminum

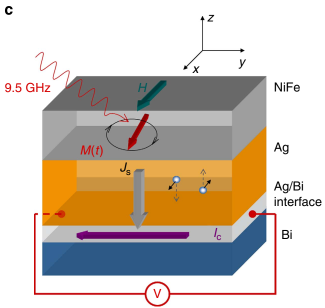


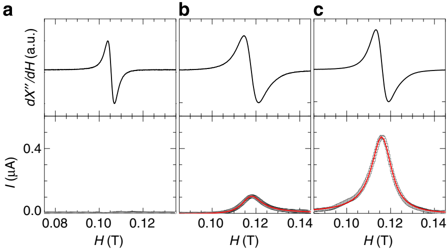


Spin current injected by ferromagnetic contact yield Hall voltage
$\sigma^{SHC} \sim 10^4 \Omega^{-1} m^{-1}$

SGE: Rojas-Sánchez et al. Nature Comm. **4**, 2944 (2013)

Metallic system: NiFe/Ag/Bi
Precessing magnetization in the ferromagnetic layer pumps spin current in the metallic bilayer Ag/Bi

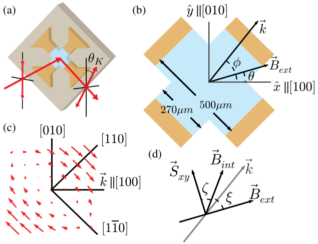


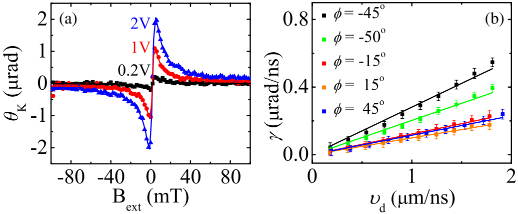


NiFe(15)/Ag(10) (a), NiFe(15)/ Bi(8) (b) and NiFe(15)/Ag(5)/Bi(8) (c)
Importance of Ag/Bi interface

ISGE: Norman et al., Phys. Rev. Lett. **112**, 056601 (2014)

Semiconducting epilayer of InGaAs





Drift current by an applied electric field generates in-plane spin polarization with rate γ.

An applied magnetic field generates an out-of-plane spin polarization measured by the Kerr rotation angle θ_K

Spin-orbit coupling: from vacuum to solids

Pauli Hamiltonian

Spin-orbit interaction arises in the non-relativistic limit of the Dirac equation

$$H_{so} = \frac{\lambda_0^2}{4}\nabla(e\phi) \times (-\mathrm{i}\nabla) \cdot \boldsymbol{\sigma},$$

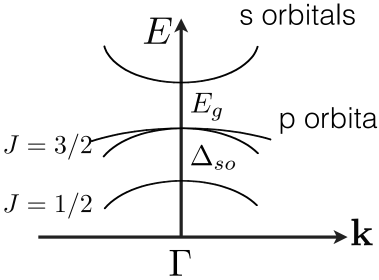
$\lambda_0 = \hbar/mc \sim 10^{-10}\mathrm{cm} \sim 10^{-2}\mathring{A}$
is the Compton wave length.

In a semiconductor the Dirac gap is replaced by the gap E_g and the effective Compton wave length can be several orders of magnitude larger

$$\frac{\lambda_0^2}{4} \equiv \frac{P^2}{3}\left[\frac{1}{E_g^2} - \frac{1}{(E_g + \Delta_{so})^2}\right]$$

$\lambda_0^2 = 5.4\mathring{A}^2$ in GaAs
$\lambda_0^2 \sim 120\mathring{A}^2$ in InAs

The Kane model for semiconductors



The valence bands of the $j = 3/2$ multiplet divide at $\mathbf{k} \neq 0$ in the heavy and light hole bands called HH and LH, respectively. The $j = 1/2$ doublet is called the split off band. In transport measurements only the HH and LH are relevant.

Extrinsic and intrinsic SOC in solids

In a solid, an electron feels the potential of the host lattice, of defects and impurities and of that induced by the confinement due an artificial structure. Potential from impurity enters the SOC Hamiltonian with the effective Compton wave length and is regarded as *extrinsic*.
In the presence of time reversal symmetry, by Kramers theorem

$$E(\mathbf{k}, \uparrow) = E(-\mathbf{k}, \downarrow).$$

If the system has space inversion symmetry

$$E(\mathbf{k}, \downarrow) = E(-\mathbf{k}, \downarrow).$$

Spin degeneracy is then present when both symmetries are satisfied.
Intrinsic SOC arises from the periodic and/or confinement potential when parity is lifted:

- 3D: lack of bulk inversion symmetry as in III-V semiconductors
- 2D: quantum wells heterostructures or interfaces

$$H_{so} = \mathbf{b}(\mathbf{k}) \cdot \boldsymbol{\sigma}$$

The Rashba spin-orbit coupling model (Sov. Phys. Solid State 2, 1109 (1960))

Emmanuel I. Rashba

(From Staff photo Matt Craig/Harvard News Office)

In his own words

"*In the late 1950s, we embarked with my student Valentin Sheka on finding the energy spectrum of uniaxial non-centrosymmetric crystals of the CdS type by group-theoretical methods, and arrived at the relativistic part of the Hamiltonian*

$$\hat{H}_{so} = \alpha(\boldsymbol{\sigma} \times \mathbf{k}) \cdot \boldsymbol{\nu}.$$

This prediction was initially met with scepticism, and my talk at the Landau Seminar was interrupted by Vitaly L. Ginzburg who exclaimed: "Because the final result is definitely wrong, there should be some error there". Landau's immediate response: "Vitya, don't you see what kind of Hamiltonian he has?"settled the problem."

The *disordered* Rashba model for the two-dimensional electron gas (2DEG)

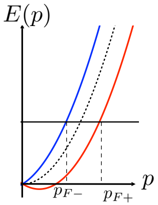
Modern era of Rashba SOC

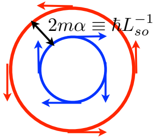
Spin degeneracy by time-reversal symmetry and parity. Breaking the parity perpendicularly to the 2DEG

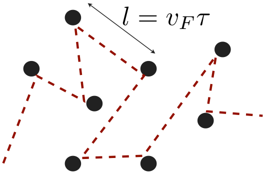
(Bychkov and Rashba, JETP Lett. **39**, 78(1984); J. Phys. C: Solid State Phys. **17**, 6093 (1984))

$$H = \frac{p^2}{2m} + \alpha(p_y\sigma^x - p_x\sigma^y) + V(\mathbf{r})$$

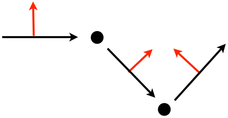
Disorder potential: $\langle V(\mathbf{r})V(\mathbf{r}')\rangle = n_i v_0^2 \delta(\mathbf{r}-\mathbf{r}')$







Random scatterers



Spin relaxation

Dyakonov-Perel

Spin precession $\omega = 2\alpha p_F$

Brownian motion in the angle space

$$\frac{1}{\tau_{DP}} \sim \frac{1}{\tau}(2\alpha p_F \tau)^2$$

A little heuristic exercise with the Boltzmann equation

Boltzmann equation for $f \equiv f(\mathbf{p}, \mathbf{r}, t)$

$$\partial_t f + \frac{\mathbf{p}}{m} \cdot \nabla_{\mathbf{r}} f - e\left(\mathbf{E} + \frac{\mathbf{p}}{m} \times \mathbf{B}\right) \cdot \nabla_{\mathbf{p}} f = I[f]$$

Before the SU(2) case, look the U(1) gauge invariance of the Boltzmann eq. with impurity scattering (collision integral $I[f]$)

Gauge invariant Keldysh Green function

After defining

$$\begin{aligned} \tilde{G}(x,p) &= -2\pi i \delta(\epsilon - \epsilon_{\mathbf{p}} + \epsilon_F) f(\mathbf{p}, \mathbf{r}, t), \\ F^{\mu\nu} &= \partial^\mu A^\nu - \partial^\nu A^\mu, \\ V^\mu &= (1, \mathbf{p}/m), \\ \epsilon_{\mathbf{p}} &= p^2/2m \end{aligned}$$

Covariant *relativistic* notation

$$\begin{aligned} x^\mu = (t, \mathbf{r}),\ & x_\mu = (-t, \mathbf{r}), \\ p^\mu = (\epsilon, \mathbf{p}),\ & p_\mu = (-\epsilon, \mathbf{p}), \\ \partial^\mu \equiv \frac{\partial}{\partial x_\mu},\ & \partial_\mu \equiv \frac{\partial}{\partial x^\mu}, \\ \partial_p^\mu \equiv \frac{\partial}{\partial p_\mu},\ & \partial_{p,\mu} \equiv \frac{\partial}{\partial p^\mu} \end{aligned}$$

Covariant form of Boltzmann eq.

$$V^\mu \partial_\mu \tilde{G} + e V^\mu F_{\mu\nu} \partial_p^\mu \tilde{G} = I[\tilde{G}]$$

Next step: generalize to SU(2)

Heuristic derivation of the quantum kinetic equation for SU(2) symmetry

the covariant derivative

This is due to the non-abelian nature of SU(2)

$$\tilde{\partial}_\mu \tilde{G} = \partial_\mu \tilde{G} + i\left[eA_\mu, \tilde{G}\right],$$
$$A^\mu = (\Phi, \mathbf{A}) = A^{\mu,0}\sigma^0 + A^{\mu,a}\frac{\sigma^a}{2}$$

the field strength

$$F^{\mu\nu} = \partial^\mu A^\nu - \partial^\nu A^\mu + ie\left[A_\mu, A_\nu\right]$$

Note: the "extra" commutator term is present also for static and uniform fields

The Su(2) covariant form

$$V^\mu \tilde{\partial}_\mu \tilde{G} + \frac{1}{2} V_\mu \left\{eF^{\mu\nu}, \partial_{p,\nu}\tilde{G}\right\} = I[\tilde{G}]$$

Anticommutator needed for ordering of operators
Gauge field appears via $\tilde{\partial}_\mu$ and $F^{\mu\nu}$
Keldysh field theory calculation confirms the above heuristic derivation

(for a recent review Raimondi et al. 2016)

Components of the SU(2) vector potential

By comparing with the Rashba Hamiltonian

$$e(A)^x_y = -e(A)^y_x = 2m\alpha,$$
$$eB^z_z = -(2m\alpha)^2$$

The way back from $\tilde{G}$ to f: the U(1) $\times$ SU(2) Boltzmann equation

Equation for the distribution function

$$\left(\tilde{\partial}_t + \frac{\mathbf{p}}{m}\cdot\tilde{\nabla}_{\mathbf{r}}\right) f(\mathbf{p},\mathbf{r},t) - \frac{1}{2}\left\{e\left(\mathbf{E}+\frac{\mathbf{p}}{m}\times\mathbf{B}\right)\cdot\nabla_{\mathbf{p}}, f(\mathbf{p},\mathbf{r},t)\right\} = I[f]$$

$$I[f] = -2\pi\int_{\mathbf{p}'} n_i v_0^2 \delta(\epsilon_{\mathbf{p}}-\epsilon_{\mathbf{p}'})(f(\mathbf{p},\mathbf{r},t)-f(\mathbf{p}',\mathbf{r},t)).$$

The fields

$$\begin{aligned}
\mathbf{E} &= -\partial_t\mathbf{A} - \nabla_{\mathbf{r}}\Phi + ie\left[\Phi,\mathbf{A}\right]\\
B_i &= \frac{1}{2}\varepsilon_{ijk}F^{jk}\\
F^{jk} &= \frac{1}{2}\varepsilon_{ijk}\left(\partial_j A_k - \partial_k A_j + i\left[A_j,A_k\right]\right)\\
& i,j,k = 1,\ldots,d.
\end{aligned}$$

Note: both **E** and **B** are still two by two matrices

Density and current matrices

$$\rho(\mathbf{r},t) = \int_{\mathbf{p}} f(\mathbf{p},\mathbf{r},t),\ \mathbf{J}(\mathbf{r},t) = \int_{\mathbf{p}}\frac{\mathbf{p}}{m}f(\mathbf{p},\mathbf{r},t).$$

Continuity-like equation

$$\tilde{\partial}_t\rho(\mathbf{r},t) + \tilde{\nabla}_{\mathbf{r}}\cdot\mathbf{J}(\mathbf{r},t) = 0.$$

Note: non conservation of spin is "hidden" in the covariant derivative

Separating the U(1) and SU(2) currents (in the diffusive approximation)

Number (Charge)

$$\mathbf{J}^0 = -\underbrace{\sigma^0(\mu)e\mathbf{E}^0}_{\text{Drude}} - \underbrace{\frac{\sigma^a(\mu)}{2}e\mathbf{E}^a}_{\text{Charge-spin coupling}} - \underbrace{D^0(\mu)\nabla_{\mathbf{r}} n}_{\text{Diffusion}} - \underbrace{2D^a(\mu)\left[\tilde{\nabla}_{\mathbf{r}} S\right]^a}_{\text{Charge-spin diffusion}}$$
$$+ \underbrace{\frac{\tau}{m}e\mathbf{B}^0\times\mathbf{J}^0}_{\text{Standard Hall}} + \underbrace{\frac{\tau}{m}e\mathbf{B}^a\times\mathbf{J}^a}_{\text{Inverse Spin Hall coefficient}}$$

Spin

$$\mathbf{J}^a = -\underbrace{\frac{\sigma^0(\mu)}{4}e\mathbf{E}^a}_{\text{Spin Drude}} - \underbrace{\frac{\sigma^a(\mu)}{2}e\mathbf{E}^0}_{\text{Charge-spin coupling}} - \underbrace{\frac{1}{2}D^a(\mu)\nabla_{\mathbf{r}} n}_{\text{Charge-spin diffusion}} - \underbrace{D^0(\mu)\left[\tilde{\nabla}_{\mathbf{r}} S\right]^a}_{\text{Covariant diffusion}}$$
$$+ \underbrace{\frac{\tau}{m}e\mathbf{B}^0\times\mathbf{J}^a}_{\text{Hall of spin currents}} + \underbrace{\frac{\tau}{4m}e\mathbf{B}^a\times\mathbf{J}^0}_{\text{Spin Hall coefficient}} .$$

Application to the disordered Rashba 2DEG

DC E_x field

$$\partial_t S^y = -\underbrace{2m\alpha J_y^z}_{\text{Covariant derivative}}$$

$$J_y^z = \underbrace{2m\alpha D S^y}_{\text{Covariant diffusion}} - \underbrace{m\tau\alpha^2 J_x}_{\text{Spin Hall coeffcient}}$$

$$= \underbrace{2m\alpha D S^y}_{\text{Vertex ladder diagram}} + \underbrace{\frac{e}{8\pi}\frac{2\tau}{\tau_{DP}}E_x}_{\text{Bare bubble diagram}}$$

In the weak disorder limit ($\tau_{DP} \to 2\tau$), the "bubble" term tends to a universal value

$$\sigma^{SHC} = \frac{e}{8\pi}$$

(Sinova et al. 2004)

Consequences

1. In the static and uniform limit $J_y^z = 0$, i.e. the spin Hall conductivity vanishes
   (Raimondi and Schwab 2005, Mischchenko et al. 2004, Inoue et al. 2004, Dimitrova 2005)
2. Spin density relaxes to the field-dependent value required by ISGE

$$\partial_t S^y = -\frac{1}{\tau_{DP}}\left(S^y + eN_0\alpha\tau E_x\right)$$

(Edelstein 1990, Aronov, Lyanda-Geller 1989; see also review by Ganichev et al. 2016))

What happens with other sources of SOC? Are they additive? Do they need to be considered?

Extrinsic SOC

Scattering amplitude in the presence of SOC

$$S_{\mathbf{pp}'} = A + B\hat{\mathbf{p}} \times \hat{\mathbf{p}}' \cdot \boldsymbol{\sigma},$$

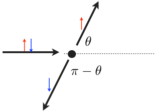
$$S^{Born}_{\mathbf{p},\mathbf{p}'} = V_{\mathbf{p}'-\mathbf{p}''}\left[1 - \frac{i\lambda_0^2}{4}\mathbf{p} \times \mathbf{p}' \cdot \boldsymbol{\sigma}\right]$$

λ_0 effective Compton wave length The density matrix changes upon scattering
$\rho_{\mathbf{p}} \rightarrow \rho_{\mathbf{p}'} = S_{\mathbf{pp}'}\rho_{\mathbf{p}}S^*_{\mathbf{pp}'}$

Various processes (Lifshits and Dyakonov 2009)

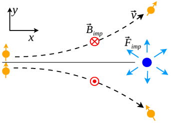
1. Standard scattering $\propto |A|^2 + |B|^2$
2. Mott skew-scattering $\propto AB^* + A^*B$
3. Spin current swapping $\propto AB^* - A^*B$
4. Spin relaxation $\propto 2|B|^2$

Mott skew scattering



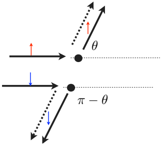
(Smit 1958)

Spin current swapping



(Lifshits and Dyakonov 2009)

Side jump



(Berger 1970)

Interplay of intrinsic and extrinsic SHE in the 2DEG

Effective Boltzmann equation up to order λ_0^2

$$\tilde{\partial}_t f_{\mathbf{p}} + \tilde{\nabla}_{\mathbf{R}} \cdot \left[\frac{\mathbf{p}}{m} f_{\mathbf{p}} + \underbrace{\frac{\lambda_0^2}{8\tau} \langle \{ \boldsymbol{\sigma} \times (\mathbf{p}' - \mathbf{p}), f_{\mathbf{p}'} \} \rangle}_{\text{from } I^{SJ}} \right] - \frac{e}{2} \left\{ \left(\mathbf{E} + \frac{\mathbf{p}}{m} \times \mathbf{B} \cdot \nabla_{\mathbf{p}} \right), f_{\mathbf{p}} \right\}$$

$$= \underbrace{-\frac{1}{\tau}\left(f_{\mathbf{p}} - \langle f_{\mathbf{p}'} \rangle\right)}_{\text{Born}} - \underbrace{(\pi v_0 N_0) \frac{\lambda_0^2}{4\tau} \langle \{ \mathbf{p}' \times \mathbf{p} \cdot \boldsymbol{\sigma}, \ f_{\mathbf{p}'} \} \rangle}_{\text{Skew scattering}} - \underbrace{i \frac{\lambda_0^2}{4\tau} \langle [\boldsymbol{\sigma} \cdot \mathbf{p} \times \mathbf{p}', f_{\mathbf{p}'}] \rangle}_{\text{spin swapping}}$$

$$+ \underbrace{\frac{\lambda_0^2}{8\tau} \langle \left(\left\{ \tilde{\nabla}_{\mathbf{r}} (\boldsymbol{\sigma} \times \mathbf{p}'), f_{\mathbf{p}'} \right\} - \left\{ \tilde{\nabla}_{\mathbf{r}} (\boldsymbol{\sigma} \times \mathbf{p}), f_{\mathbf{p}} \right\} \right) \rangle}_{\text{from } I^{SJ}\text{: to ensure that } f_{eq}(\epsilon_{\mathbf{p}}) \text{ be a solution}}$$

$$f_{\mathbf{p}} = f(\epsilon_{\mathbf{p}}, \hat{\mathbf{p}}), \quad f_{\mathbf{p}'} = f(\epsilon_{\mathbf{p}}, \hat{\mathbf{p}}'), \quad \text{elastic scattering}$$

$$\tilde{\nabla}_{\mathbf{r}} \equiv \nabla_{\mathbf{r}} - e\mathbf{E}^0 \partial_{\epsilon_{\mathbf{p}}} + i\,[e\mathbf{A}, \ldots] \quad \text{covariant derivative}$$

$$\langle \ldots \rangle \equiv \int \frac{\mathrm{d}\Omega_{\mathbf{p}'}}{\Omega} \ldots, \quad \text{integration over } \hat{\mathbf{p}}'$$

When intrinsic SOC vanishes, one recovers the extrinsic SOC results

Noziè and Lewiner J. Phys. **34**, 901 (1973);Engel et al. PRL **95**, 166605 (2005); Tse and Das Sarma PRL **96**, 056601 (2006)

When both intrinsic and extrinsic SOC: apparent paradox

Tse and Das Sarma PRB **74**, 245309 (2006); Hankiewicz and Vignale PRL **100**, 026602 (2008)

DC E_x field

$$\partial_t S^y = -\underbrace{2m\alpha J_y^z}_{\text{Covariant derivative}} \quad \text{The current includes the s.j. correction}$$

$$J_y^z = \underbrace{2m\alpha D S^y}_{\text{Anomalous diffusion}} + \underbrace{\frac{e}{8\pi}\frac{2\tau}{\tau_{DP}}E_x}_{\text{Intrinsic}} + \underbrace{(\sigma_{ss}^{SHC} + \sigma_{sj}^{SHC})E_x}_{\text{Extrinsic}},$$

Consequences

1. In the static and uniform limit $J_y^z = 0$, i.e. the spin Hall conductivity vanishes.
2. Non analytical behavior

$$\lim_{\alpha\to 0} \sigma^{SHC}(\alpha, \lambda_0) \neq \sigma^{SHC}(0, \lambda_0)$$

How to avoid it?

Interplay of intrinsic and extrinsic SHE in the 2DEG

Effective Boltzmann equation up to order λ_0^4

$$\tilde{\partial}_t f_{\mathbf{p}} + \tilde{\nabla}_{\mathbf{R}} \cdot \left[\frac{\mathbf{p}}{m} f_{\mathbf{p}} + \underbrace{\frac{\lambda_0^2}{8\tau} \langle \{ \boldsymbol{\sigma} \times (\mathbf{p}' - \mathbf{p}), f_{\mathbf{p}'} \} \rangle}_{\text{from } I^{SJ}} \right] - \frac{e}{2} \left\{ \left(\mathbf{E} + \frac{\mathbf{p}}{m} \times \mathbf{B} \cdot \nabla_{\mathbf{p}} \right), f_{\mathbf{p}} \right\}$$

$$= \underbrace{-\frac{1}{\tau} \left(f_{\mathbf{p}} - \langle f_{\mathbf{p}'} \rangle \right)}_{\text{Born}} \underbrace{- (\pi v_0 N_0) \frac{\lambda_0^2}{4\tau} \langle \{ \mathbf{p}' \times \mathbf{p} \cdot \boldsymbol{\sigma}, \; f_{\mathbf{p}'} \} \rangle}_{\text{Skew scattering}} \underbrace{- i \frac{\lambda_0^2}{4\tau} \langle [\boldsymbol{\sigma} \cdot \mathbf{p} \times \mathbf{p}', f_{\mathbf{p}'}] \rangle}_{\text{spin swapping}}$$

$$\underbrace{+ \frac{\lambda_0^2}{8\tau} \langle \left(\left\{ \tilde{\nabla}_{\mathbf{r}} (\boldsymbol{\sigma} \times \mathbf{p}'), f_{\mathbf{p}'} \right\} - \left\{ \tilde{\nabla}_{\mathbf{r}} (\boldsymbol{\sigma} \times \mathbf{p}), f_{\mathbf{p}} \right\} \right) \rangle}_{\text{from } I^{SJ}\text{: to ensure that } f_{eq}(\epsilon_{\mathbf{p}}) \text{ be a solution}}$$

$$\underbrace{- \frac{\lambda_0^4}{16} 2\pi n_i v_0^2 \sum_{\mathbf{p}'} \delta(\epsilon_{\mathbf{p}} - \epsilon_{\mathbf{p}'}) \left[f_{\mathbf{p}} - \sigma^z f_{\mathbf{p}'} \sigma^z \right] (\mathbf{p} \times \mathbf{p}')_z^2}_{\text{Elliott-Yafet spin relaxation } \propto |B|^2}$$

Raimondi and Schwab EPL **87**, 37008; Raimondi et al. Ann. Phys. **524**, 153 (2012); Shen et al. PRB **90**, 245302 (2014)

Solution of the paradox

$$\frac{1}{\tau_{DP}} = (2m\alpha)^2 D$$
$$\frac{1}{\tau_{EY}} = \frac{1}{\tau}\left(\frac{\lambda_0 k_F}{2}\right)^4$$

With no intrinsic SOC, the evaluation of the spin Hall conductivity can be carried out to order λ_0^2, neglecting higher order corrections. When intrinsic SOC is present, it is important the relative size of the two relaxation channels. One must considers terms of order λ_0^4, where extrinsic SOC spin relaxation first occurs.

DC E_x field

$$\partial_t S^y = -2m\alpha J_y^z - \frac{1}{\tau_{EY}} s^y$$

$$J_y^z = 2m\alpha D S^y + (\sigma_{int}^{SHC} + \sigma_{ext}^{SHC})E_x,$$
$$\sigma_{ext}^{SHC} = \sigma_{sj}^{SHC} + \sigma_{ss}^{SHC}$$

Spin Hall and spin Galvanic effect

$$J_y^z = \frac{1}{1+\tau_{EY}/\tau_{DP}}(\sigma_{int}^{sH} + \sigma_{ext}^{SHC})E_x$$
$$s^y = -\frac{2m\alpha}{1/\tau_{EY} + 1/\tau_{DP}}(\sigma_{int}^{SHC} + \sigma_{ext}^{SHC})E_x,$$

No paradox: when $\alpha \to 0$, $\tau_{DP} \to \infty$ and $\sigma_{int}^{SHC} \to 0$

Consequences

- In the Rashba model for the 2DEG, the interplay of intrinsic and extrinsic SOC is not additive and the strength of the SHE can be controlled by the relative size of the SOC mechanisms
- The SGE and ISGE is closely related to the SHE. Although the Rashba SOC is still an essential ingredient, the interplay of intrinsic and extrinsic SOC yields a value depending on total spin relaxation and SHE

$$S^y = -2m\alpha\tau_s\sigma^{SHC}$$

- More importantly, when also Dresselhaus is present, the induced spin polarization is in general different from the direction of the internal anisotropic (in momentum space) "magnetic" field. This is compatible with recent measurements in semiconducting thin films

Norman et al. PRL **112**, 056601 (2014)

- SGE, the charge current induced by a the injected spin polarization as in a spin-pumping experiment, can seen a two-step process: 1) spin current is created by the spin accumulation due to pumped spin current; 2) charge current is created by ISHE; $J_y^z = 2m\alpha D\tau_s J_s^y \Rightarrow J_x = -\gamma J_y^z$

$$\lambda_{IEE} = \alpha\tau_s \frac{\sigma^{SHC}}{e/8\pi}$$

Shen et al. PRL **112**, 096601 (2014); Rojas Sánchez et al. Nat. Commun. **4**, 2944 (2013)

Take-home messages again

1. Spin-charge conversion mechanisms in transport phenomena are studied in a large variety of systems ranging from semiconductors (GaAs 2DEGs), metals (FM/AgBi) and insulators (oxides interfaces LAO/STO). Optical (Kerr and Faraday spectroscopies) and spin pumping (FMR) techniques are used together with the standard electric-field induced excitation of carriers.
2. The general features of these phenomena can be understood by appealing to symmetry arguments analyzing in particular time reversal and parity. Spin degeneracy is a result of both symmetries. Internal magnetic fields arise when parity is broken.
3. The microscopic mechanism responsible for spin-charge conversion is the spin-orbit coupling (SOC) and we talk today of Spin-orbitronics. SOC in solid state systems manifests in a number of ways. Effective Compton wave length up to three orders of magnitude bigger. Its description in the two-dimensional electron gas can be formulated in an elegant way as an effective SU(2) gauge theory formulation of the familiar Boltzmann transport equation via non-equilibrium quantum field theory.

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Thank you
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